

Task Displacement and Wage Inequality

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Rising Wage Inequality Between Groups of Society

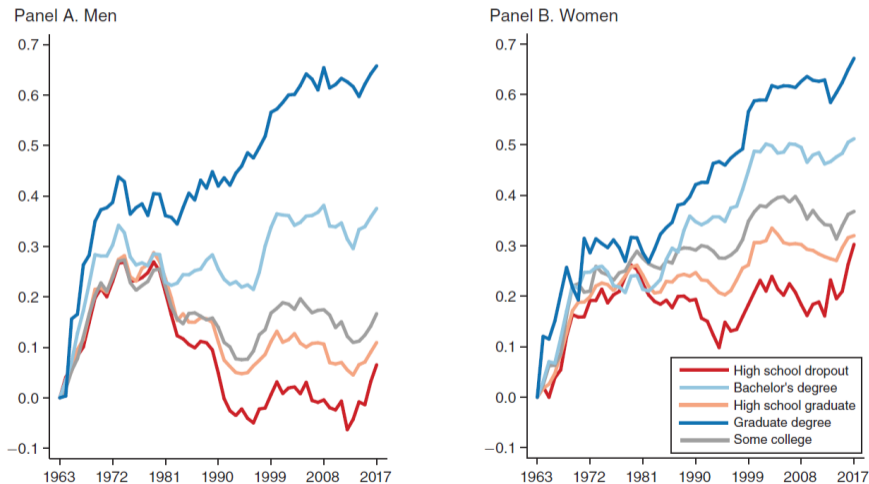


Figure: Cumulative change in real weekly wages, working-age adults (Autor, 2019)

Rising Wage Inequality Between Groups of Society

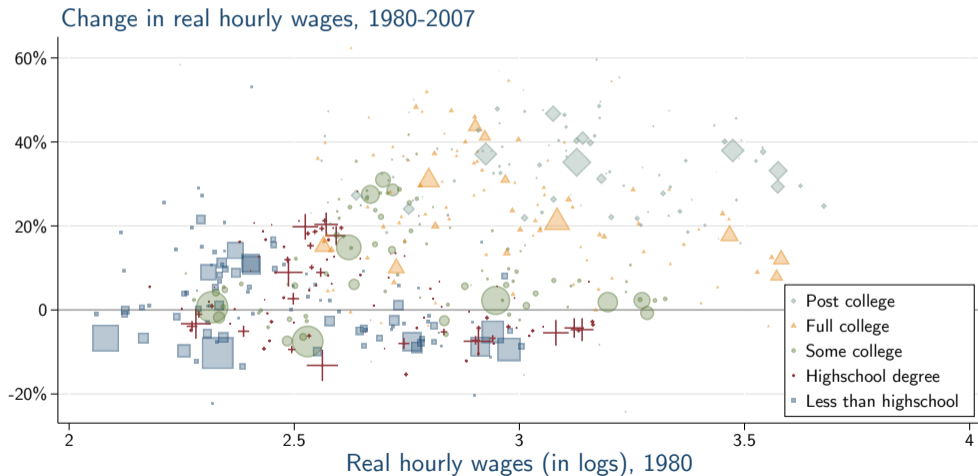


Figure: Change in real hourly wages for 500 education–experience–gender–race–nativity groups

Rising Wage Inequality Between Groups of Society

Task-displacing technologies \Rightarrow wage inequality across groups?

\Rightarrow stagnant or declining wages?

This paper

- Task framework: wages depend on **allocation of tasks** to workers (Grossman–Rossi-Hansberg 2008; Acemoglu–Autor 2011; Acemoglu–Restrepo 2018)
- automation and offshoring **change boundaries** of allocation
- **quantify** role of **task-displacement** via automation and offshoring

Existing literature

- SBTC (Katz–Murphy 92; Krusell–et.al 00; Card–Lemieux 01)
- industry shifts and demand for skills (Buera–et.al 15; Bárány–Siegel 18)
- occupational shifts (Lee–Shin 18; Jaimovich–et.al 20)

Outline of the Paper

Tractable task framework

- role of task allocation $\ln w_g = a \cdot \ln(y/\ell_g) + b \cdot \ln \text{task share}_g$
- automation and offshoring \Rightarrow change $\ln \text{task share}_g$ and tfp
- large distributional effects and small tfp gains $\Rightarrow d \ln w_g < 0$

Measure task displacement & reduced forms

- $\text{task displacement}_g$ = effect of technology on $\ln \text{task share}_g$
- g displaced from **routine tasks** in industry with falling **labor share**
- correlates with wage changes across groups

Quantifying effect of task displacement

- use model to compute effects on output and wages
- account for **ripple effects**, **industry shifts** and **productivity gains**
- explain 48% to 57% of wage changes and sizable share of declines

Outline of the Talk

1. Task model with multiple skills
 - effect of technology on wages and tfp
 - model with multiple industries to connect with data
2. Measuring task displacement
 - and reduced-form evidence
3. Quantifying effect of task displacement on wages and tfp

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Model: Environment

Output combines
mass M of tasks in \mathcal{T}

$$y = \left(\frac{1}{M} \int_{\mathcal{T}} (M \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}}, \quad \lambda = \text{task subs.}$$

Tasks produced by
capital or different
types of labor g

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_g A_g \cdot \psi_g(x) \cdot \ell_g(x).$$

Factor supply and
equilibrium

▶ formal definition

- capital $k(x)$ produced from final good at rate $r \cdot q(x)$
- labor of type g has fixed supply $\ell_g > 0$
- allocation of tasks to factors maximizes $y - r \cdot \int_{\mathcal{T}} k(x) \cdot q(x) \cdot dx$

Model: Allocation of Tasks and Task Shares

Task allocation
defined by sets
 \mathcal{T}_g and \mathcal{T}_k

$$\mathcal{T}_g := \left\{ x : \frac{1}{\psi_g(x)} \cdot \frac{w_g}{A_g} \leq \frac{1}{\psi_j(x)} \cdot \frac{w_j}{A_j}, \frac{q(x)}{\psi_k(x)} \cdot \frac{r}{A_k} \quad \forall j \right\}$$

$$\mathcal{T}_k := \left\{ x : \frac{q(x)}{\psi_k(x)} \cdot \frac{r}{A_k} \leq \frac{1}{\psi_j(x)} \cdot \frac{w_j}{A_j} \quad \forall j \right\}$$

Definition of
task share of g
& task share k

$$\Gamma_g(w^e, \Psi) := \frac{1}{M} \int_{\mathcal{T}_g} \psi_g(x)^{\lambda-1} \cdot dx$$

$$\Gamma_k(w^e, \Psi) := \frac{1}{M} \int_{\mathcal{T}_k} (\psi_k(x)/q(x))^{\lambda-1} \cdot dx.$$

Determinants
of Γ_g and Γ_k

- wages/rates per efficiency unit $w^e = \{w_1/A_1, \dots, w_G/A_G, c/A_k\}$.
- task-specific technologies $\Psi \Rightarrow$ also affect boundaries $\mathcal{T}_g, \mathcal{T}_k!$

Proposition (Equilibrium objects as function of task shares)

Given $\ell = (\ell_1, \ell_2, \dots, \ell_G)$ and task shares $\{\Gamma_1, \dots, \Gamma_G, \Gamma_k\}$, output is given by

$$y = (1 - (c/A_k)^{1-\lambda} \cdot \Gamma_k)^{\frac{\lambda}{1-\lambda}} \cdot \left(\sum_g \Gamma_g^{\frac{1}{\lambda}} \cdot (A_g \cdot \ell_g)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \quad (1)$$

wages are given by

$$w_g = \left(\frac{y}{\ell_g} \right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}}. \quad (2)$$

and factor shares are given by

$$s^K = (r/A_k)^{1-\lambda} \cdot \Gamma_k, \quad s^L = 1 - (r/A_k)^{1-\lambda} \cdot \Gamma_k. \quad (3)$$

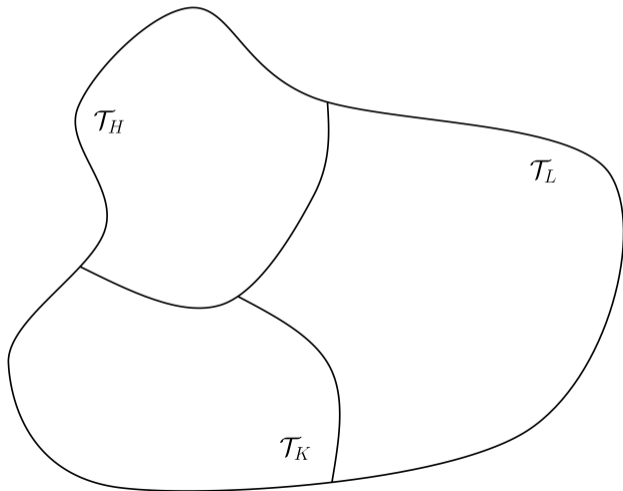
Model: A Wide Menu of Technologies

Conditional on w^e , two main technology classes changing Γ_g and Γ_k :

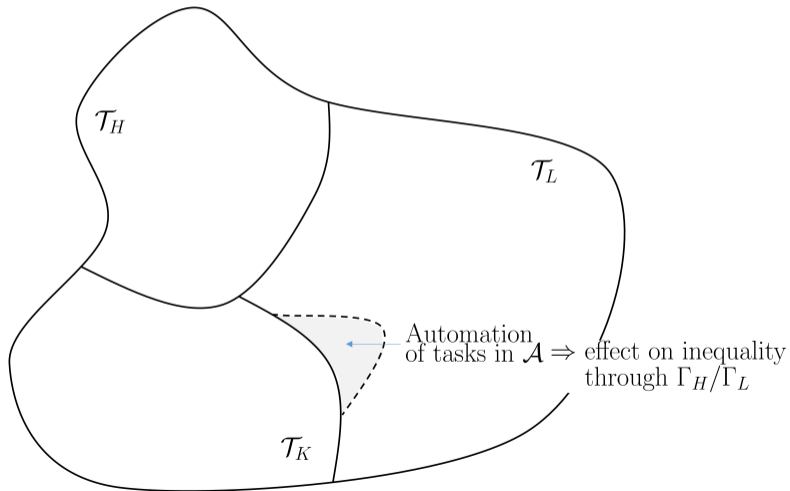
Productivity deepening	<ul style="list-style-type: none">▪ improvements in $\psi_g(x)$ or $\psi_k(x)/q(x)$ for tasks in \mathcal{T}_g or \mathcal{T}_k▪ denote effect on $\frac{1}{1-\lambda}d\ln \Gamma_g$ by $d\ln \Gamma_g^{\text{deep}}$ or $d\ln \Gamma_k^{\text{deep}}$
Task displacement via automation or offshoring	<ul style="list-style-type: none">▪ $\mathcal{T}_g \downarrow$ and $\mathcal{T}_k \uparrow$ due to improvements in $\psi_k(x)/q(x)$ for tasks in \mathcal{T}_g▪ denote effect on $d\ln \Gamma_g$ by $d\ln \Gamma_g^{\text{disp}}$▪ $\pi_g = \text{avg cost reduction in such tasks } \ln w_g/\psi_g(x) - \ln \psi_k(x)/q(x)$

Besides: usual factor augmenting technologies, A_g and A_k , affect task shares through w^e .

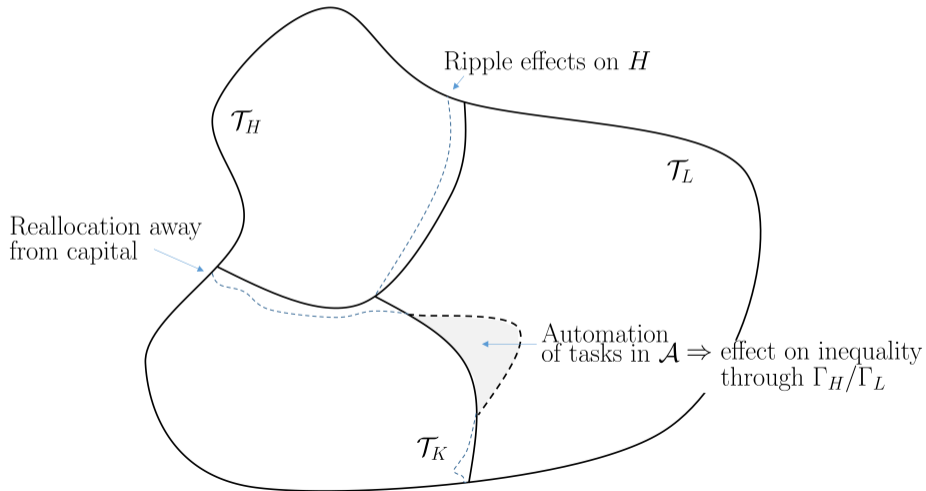
Model: Examples of Different Technologies



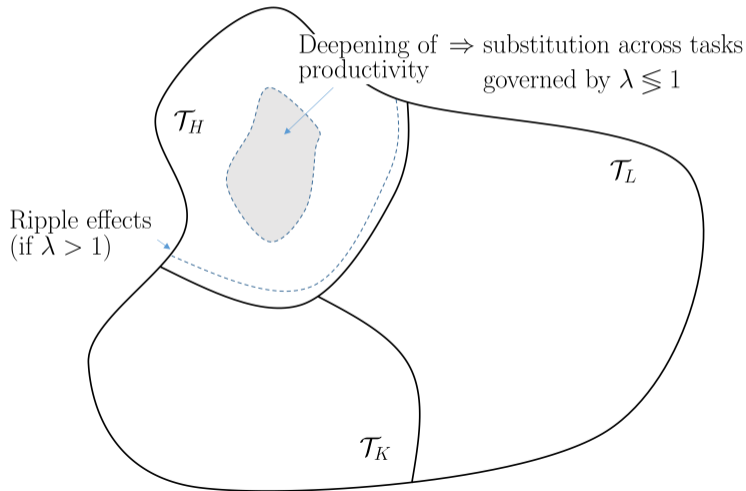
Model: Examples of Different Technologies



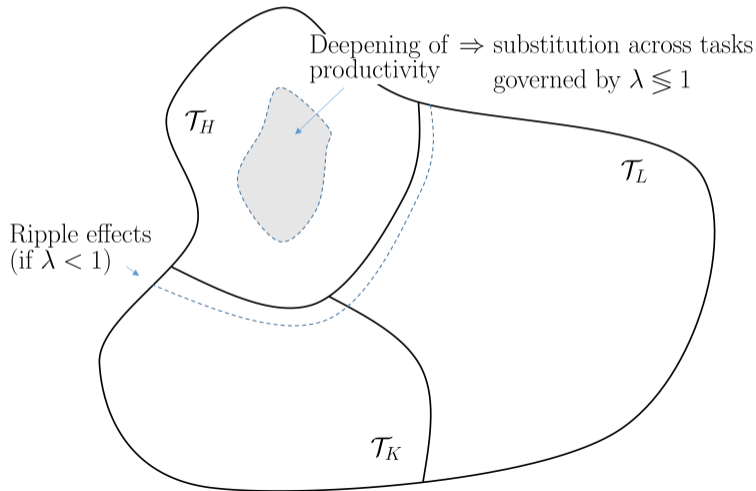
Model: Examples of Different Technologies



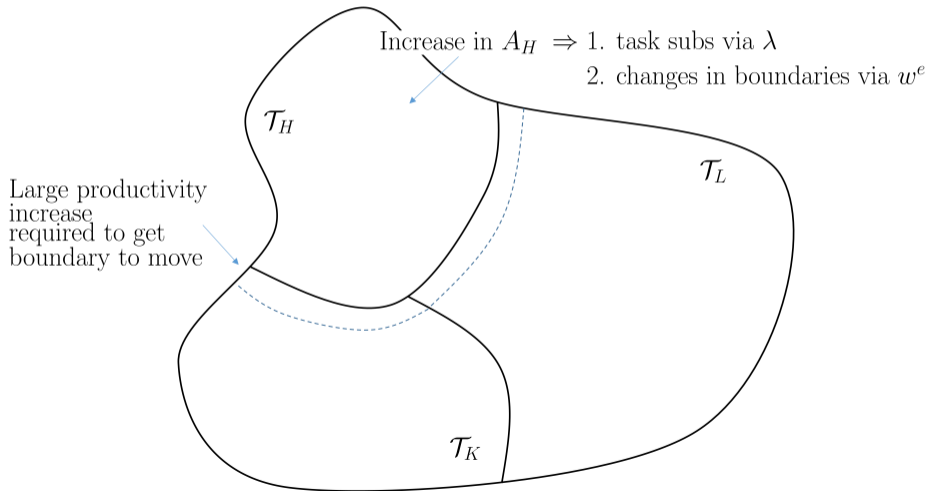
Model: Examples of Different Technologies



Model: Examples of Different Technologies



Model: Examples of Different Technologies



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Effects of Technology: Shock and Propagation

Propagation of
a wage shock

- $d \ln w_g = a_g + \frac{1}{\lambda} \frac{\partial \ln \Gamma_g}{\partial \ln w^e} \cdot d \ln w \Rightarrow d \ln w = \Theta \cdot a_L$, where

$$\Theta := \left(\mathbb{1} - \frac{1}{\lambda} \frac{\partial \ln \Gamma_L(w^e; \Psi)}{\partial \ln w^e} \right)^{-1} = \mathbb{1} + \frac{1}{\lambda} \frac{\partial \ln \Gamma_L}{\partial \ln w^e} + \left(\frac{1}{\lambda} \frac{\partial \ln \Gamma_L}{\partial \ln w^e} \right)^2 + \dots$$

Properties of
propagation
matrix Θ

- Θ is a $G \times G$ matrix where ripple effect of j on g is $\theta_{gj} \geq 0$
- row sum equals $\frac{\lambda}{\lambda + \varrho_g} \Rightarrow$ elast. of subs with capital $\lambda + \varrho_g$
- if $\varrho_g = \varrho$, matrix has symmetry property $s_g^L \cdot \theta_{gj} = s_j^L \cdot \theta_{jg}$
- also, g and j are q -substitutes iff $\theta_{gj} > s_j^L \cdot \lambda / (\lambda + \varrho)$
- ripple effects can dampen or augment inequality

Proposition (Effect of technology on wages and TFP)

The change in wages is given by

$$d\ln w_g = \frac{1}{\lambda + \varrho_g} d\ln y + d\ln A_g - \frac{1}{\lambda} \Theta_g \cdot d\ln A_L + \frac{\lambda - 1}{\lambda} \Theta_g \cdot d\ln \Gamma_L^{deep} + \frac{1}{\lambda} \Theta_g \cdot d\ln \Gamma_L^{disp},$$

and the change in aggregate TFP and output is given by

$$d\ln tfp = \sum_g s_g^L \cdot d\ln A_g + s^K \cdot d\ln \Gamma_k^{deep} + \sum_g s_g^L \cdot d\ln \Gamma_g^{deep} - \sum_g s_g^L \cdot d\ln \Gamma_g^{disp} \cdot \pi_g$$

$$d\ln y = \frac{1}{1 - s^K} \cdot (d\ln tfp + s^K \cdot d\ln s^K).$$

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Model: Multiple Industries

Industry
structure

- $y = \text{GDP}$ and $p = \text{vector of industry prices}$
- $s_i^Y(p, y) := \text{share industry } i \text{ in value added} \Rightarrow \text{CES } s_i^Y(p, y) = \alpha_i \cdot p_i^{1-\eta}$

Industry i
combines M_i
tasks in \mathcal{T}_i

$$y_i = A_i \cdot \left(\frac{1}{M_i} \int_{\mathcal{T}_i} (M_i \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}}, \quad \lambda = \text{task subs.}$$

Task shares
now given by

$$\Gamma_g(\zeta, w^e, \Psi) := \sum_i \underbrace{s_i^Y(p, y) \cdot (A_i \cdot p_i)^{\lambda-1}}_{:= \zeta_i} \cdot \underbrace{\frac{1}{M_i} \int_{\mathcal{T}_{gi}} \psi_g(x)^{\lambda-1} dx}_{:= \Gamma_{gi}}$$

Proposition (Equilibrium objects as function of task shares)

Given $\ell = (\ell_1, \ell_2, \dots, \ell_G)$ and within industry task shares $\{\Gamma_{1i}, \dots, \Gamma_{Gi}, \Gamma_{ki}\}$ for all i , equilibrium wages, industry prices, and output are the solution to

$$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot A_g^{\frac{\lambda-1}{\lambda}} \cdot \left(\sum_i s_i^Y(p, y) \cdot (A_i p_i)^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{\lambda}} \quad (4)$$

$$A_i p_i = \left(A_k^{\lambda-1} \cdot \Gamma_{ki} + \sum_g w_g^{1-\lambda} \cdot A_g^{\lambda-1} \cdot \Gamma_{gi}\right)^{\frac{1}{1-\lambda}} \quad (5)$$

$$1 = \sum_i s_i^Y(p, y). \quad (6)$$

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Measuring Task Displacement: Cobb-Douglas case $\lambda = 1$ and $\rho_i = 0$

A1. Technology and markups

- changes in $\psi_k(x)/q(x)$ leading to task displacement, $d \ln \Gamma_L^{\text{disp}}$
- no change in markups

A2. Routine tasks in industry i automated at common rate

- $\Gamma_{gi} = \Gamma_{gi}^N + \Gamma_{gi}^R$
- $d \ln \Gamma_{gi}^{N,\text{disp}} = 0$ and $d \ln \Gamma_{gi}^{R,\text{disp}} = d \ln \Gamma_i^{R,\text{disp}}$

A1+A2: recover task displacement from industry data on labor shares, s_i^L

$$d \ln \Gamma_i^{R,\text{disp}} = \frac{1}{s_i^R} \cdot d \ln s_i^L$$

$$d \ln \Gamma_g^{\text{disp}} = \sum_i \frac{s_{gi}^R}{s_i^R} \cdot d \ln s_i^L$$

Measuring Task Displacement: CES case

A1. Set of technologies is restricted

A2. Routine tasks in industry i automated at common rate

- sectoral productivity shocks: A_i
- capital deepening: uniform decline in $q(x)$ of $d \ln q_i$
- changes in $\psi_k(x)/q(x)$ leading to task displacement, $d \ln \Gamma_L^{\text{disp}}$
- no change in markups

- $\Gamma_{gi} = \Gamma_{gi}^N + \Gamma_{gi}^R$
- $d \ln \Gamma_{gi}^{N,\text{disp}} = 0$ and $d \ln \Gamma_{gi}^{R,\text{disp}} = d \ln \Gamma_i^{R,\text{disp}}$

A1+A2: recover task displacement from industry data on labor shares, s_i^L

$$d \ln \Gamma_i^{R,\text{disp}} = \frac{1}{s_i^R} \frac{d \ln s_i^L + (1 - \sigma_i) \cdot s_i^K \cdot (d \ln q_i - d \ln w_i)}{1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i}$$

$$d \ln \Gamma_g^{\text{disp}} = \sum_i \frac{s_{gi}^R}{s_i^R} \cdot \frac{d \ln s_i^L + (1 - \sigma_i) \cdot s_i^K \cdot (d \ln q_i - d \ln w_i)}{1 + (\lambda - 1) \cdot s_i^L \cdot \pi_i}$$

Data and Measurement

Data for 49 industries
from BEA, BLS, and
KLEMS

- for reduced form: $\sigma_i = \sigma \in (0.5, 1.2)$, $\lambda = 0.5$, $\pi_i = 30\%$
- measure task displacement from 1987-2016
- across industries, $d \ln \Gamma_i^{\text{disp}}$ correlates with:
 - ✓ rising tfp and quantities; falling prices
 - ✓ higher demand for skilled workers
 - ✓ proxies of automation and offshoring

Construct measure of
task displacement for
500 skill groups

- Census data for 1980 to measure occupational wage shares
- groups defined by education–experience–gender–race–nativity
- routine jobs measured using ONET as in Acemoglu–Autor 2011

Data and Measurement

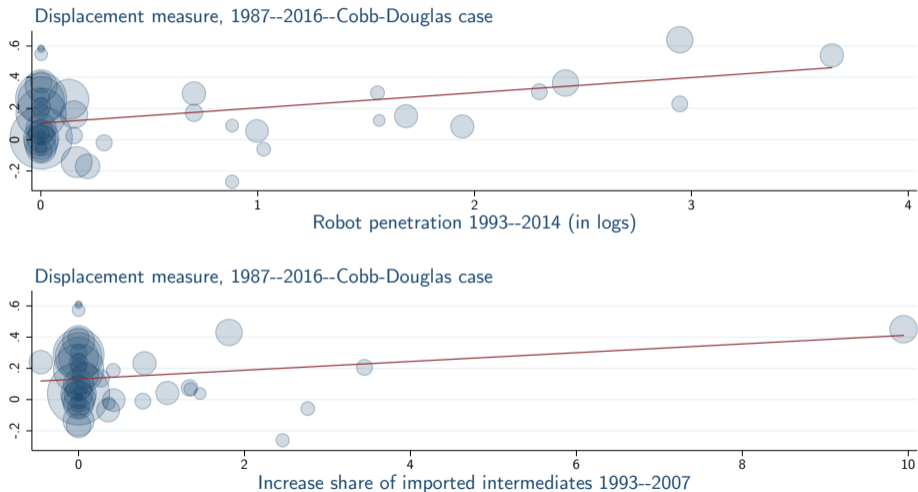


Figure: Industry correlations between proxies of automation and offshoring and task displacement

Data and Measurement

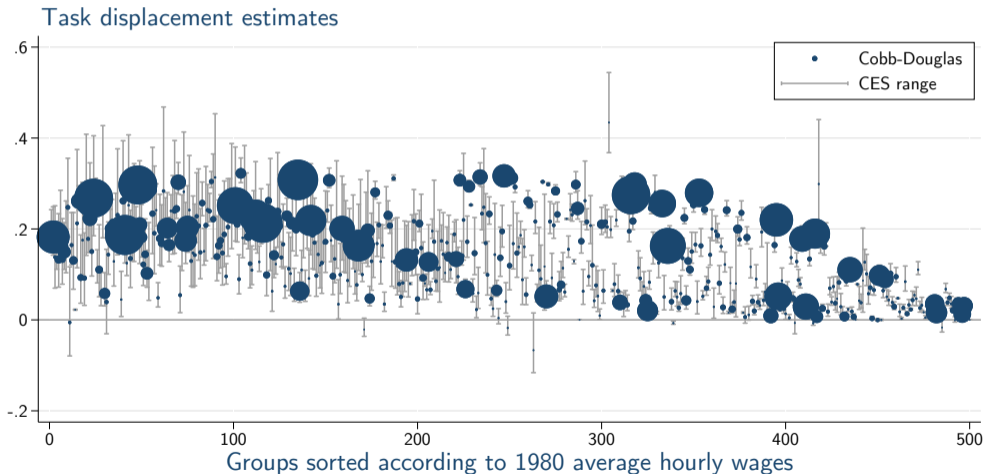


Figure: Estimated task displacement for 500 education–experience–gender–race–nativity groups

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Reduced-form evidence: Cobb-Douglas

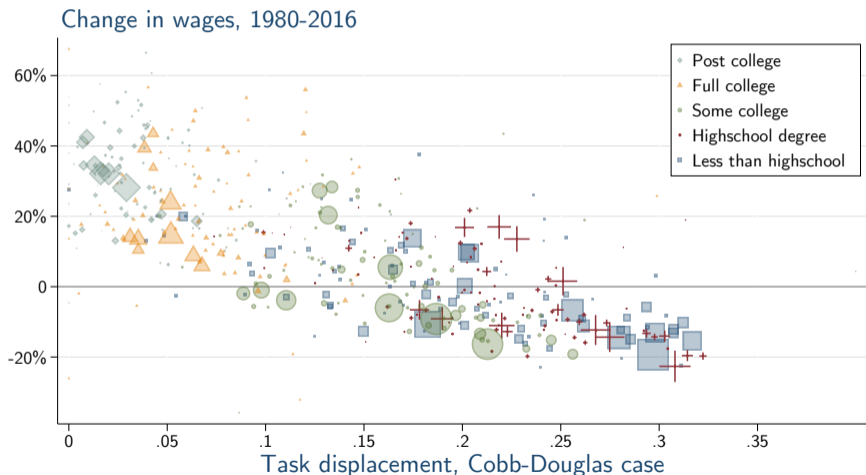


Figure: Reduced-form relation between task displacement and change in wages, 1980–2016.

Reduced-form evidence: Cobb-Douglas

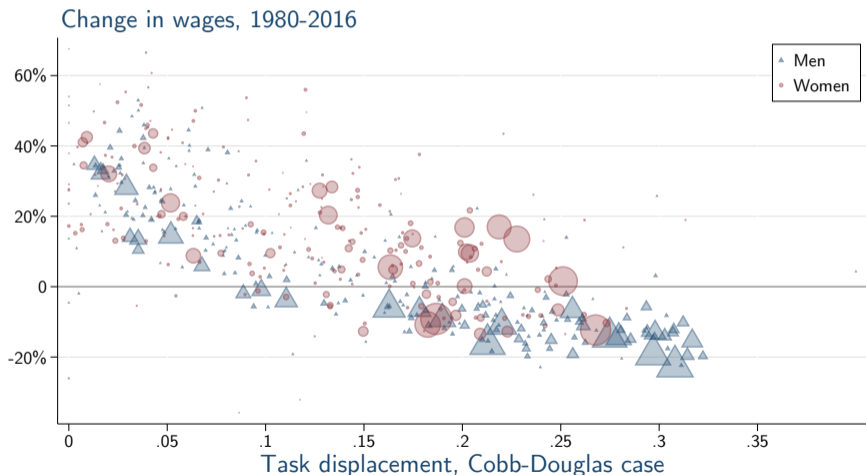


Figure: Reduced-form relation between task displacement and change in wages, 1980–2016.

Reduced-form evidence: CES

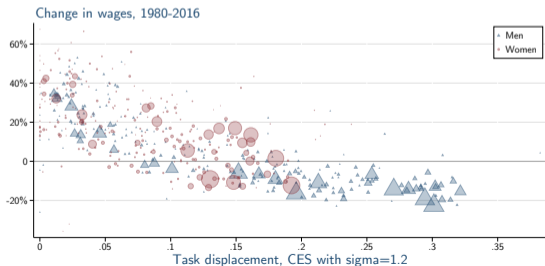
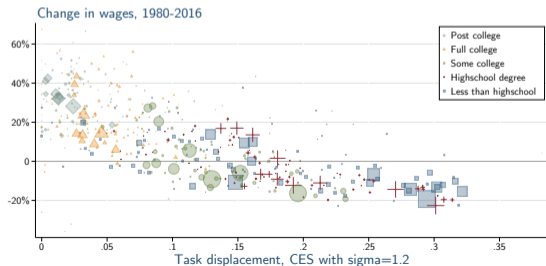
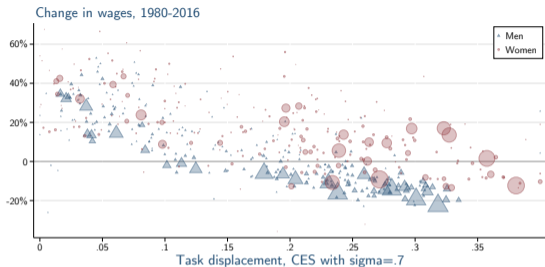
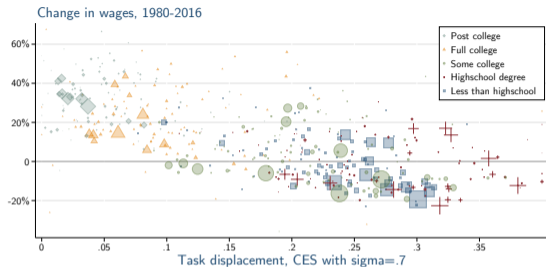


Figure: Reduced-form relation between task displacement and change in wages, 1980–2016.

Reduced-form evidence: Cobb-Douglas

	Dependent variable: change log hourly wages 1980-2016					
	(1)	(2)	(3)	(4)	(5)	(6)
Task displacement	-1.482 (0.096)	-1.132 (0.162)	-1.429 (0.302)	-1.243 (0.219)	-1.172 (0.218)	-1.032 (0.205)
Sectoral expansion		0.214 (0.076)	0.099 (0.084)	0.111 (0.079)	0.117 (0.075)	0.652 (0.155)
Industries with declining labor share			-0.416 (0.404)			
Relative specialization in routine jobs			0.060 (0.059)			
R-squared	0.62	0.66	0.70	0.77	0.79	0.81
Observations	500	500	500	500	500	500
<i>Additional covariates:</i>						
Broad group dummies				✓	✓	✓
Regional shares					✓	✓
Broad sectoral shares						✓

Technology or Rising Markups?

Industry correlates
suggest technology
important

- task displacement correlates with rising tfp and quantities, lower prices
- within manufacturing, task displacement correlates with automation and offshoring
- labor share declines mostly in manufacturing and industry

Reduced-form
evidence

- as labor share declines, labor demand falling for routine workers but not for others

Takeaway

- markups might be important, but one needs a richer theory of their relationship to tfp, technology and demand for skills

Reduced-form evidence: Task displacement vs SBTC

	Dependent variable: change log hourly wages 1980-2016		
	(1)	(2)	(3)
Education: highschool	0.005 (0.032)	0.017 (0.028)	0.027 (0.028)
Education: some college	0.032 (0.035)	-0.047 (0.037)	-0.054 (0.039)
Education: full college	0.247 (0.029)	0.030 (0.053)	-0.045 (0.060)
Education: more than college	0.395 (0.027)	0.142 (0.056)	0.016 (0.074)
Gender: women	0.144 (0.026)	0.104 (0.022)	0.070 (0.023)
Task displacement		-1.174 (0.195)	-1.032 (0.205)
Sectoral expansion			0.652 (0.155)
R-squared	0.68	0.76	0.81
Observations	500	500	500
<i>Additional covariates:</i>			
Regional and broad sectoral shares			✓

Reduced-form evidence: Robustness—CES case

	Dependent variable: change log hourly wages 1980-2016					
	$\sigma = 0.7$	$\sigma = 0.8$	$\sigma = 0.9$	$\sigma = 1$	$\sigma = 1.1$	$\sigma = 1.2$
	(1)	(2)	(3)	(4)		
Education: highschool	0.045 (0.028)	0.041 (0.029)	0.035 (0.029)	0.027 (0.028)	0.016 (0.028)	0.004 (0.027)
Education: some college	-0.004 (0.033)	-0.019 (0.035)	-0.036 (0.037)	-0.054 (0.039)	-0.072 (0.040)	-0.088 (0.040)
Education: full college	0.033 (0.054)	0.008 (0.056)	-0.019 (0.058)	-0.045 (0.060)	-0.067 (0.060)	-0.082 (0.059)
Education: more than college	0.113 (0.068)	0.082 (0.070)	0.049 (0.072)	0.016 (0.074)	-0.013 (0.074)	-0.033 (0.073)
Gender: women	0.144 (0.020)	0.124 (0.020)	0.099 (0.021)	0.070 (0.023)	0.038 (0.027)	0.007 (0.030)
Task displacement	-0.736 (0.181)	-0.841 (0.192)	-0.943 (0.201)	-1.032 (0.205)	-1.095 (0.203)	-1.120 (0.194)
Sectoral expansion	0.542 (0.164)	0.566 (0.161)	0.603 (0.158)	0.652 (0.155)	0.711 (0.153)	0.775 (0.153)
R-squared	0.79	0.80	0.81	0.81	0.81	0.82
Observations	500	500	500	500	500	500
<i>Additional covariates:</i>						
Region and broad sector shares	✓	✓	✓	✓	✓	✓

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Proposition (Counterfactuals)

The effect of task displacement by automation and offshoring on wages, industry prices and GDP is given by the solution to the following system of linear equations:

$$d \ln w_g = \frac{1}{\lambda + \varrho_g} \cdot d \ln y + \frac{1}{\lambda} \Theta_g \cdot d \ln \zeta + \frac{1}{\lambda} \Theta_g \cdot d \ln \Gamma_L^{disp},$$

$$d \ln \zeta_g = \sum_i s_{gi}^L \cdot (d \ln s_i^Y(p, y) + (\lambda - 1) \cdot d \ln p_i),$$

$$d \ln p_i = s_i^L \cdot \sum_g s_{ig}^L \cdot (d \ln w_g + d \ln \Gamma_{gi}^{disp} \cdot \pi_{gi})$$

$$d \ln tfp = - \sum_g s_g^L \sum_i s_{gi}^L \cdot d \ln \Gamma_{gi}^{disp} \cdot \pi_{gi}$$

$$d \ln y = \frac{1}{1 - s^K} \cdot (\lambda \cdot d \ln tfp - s^K \cdot d \ln s^K).$$

Estimating Θ and ϱ : Approach

- Assume $\varrho_g = \varrho \Rightarrow$ common elast of subs between capital and labor (see Dvorkin–Monge-Naranjo 2019 for approach with dif ϱ_g)
- $\beta_{gj} = \frac{1}{\lambda} \cdot \theta_{gj}/s_j^l$ is the per unit ripple effect from j to $g \Rightarrow \beta_{gj} = \beta_{jg}$
- Parametric assumption: $\theta_{gg} = \beta_{\text{own}} \geq 0$ and if $g \neq j$

$$\beta_{gj} = \sum_{n=1}^N \beta_n \cdot \exp(-d(x_g^n, x_j^n)), \text{ with } \beta_n \geq 0,$$

where x_g^n are vectors of industry shares in 1980, occupational shares in 1980, state shares in 1980 and skill level

- Combine labor supply shocks (demographic trends), sectoral shifts (Bartik measure), and task displacement into a single shock to estimate β_{own} and $\beta_n \Rightarrow \hat{\Theta}$

Estimating Θ and ϱ : Results and Parametrization

- evidence of ripple effects among:
 - groups in similar industries
 - groups in similar occupations
 - groups in similar states
 - groups of similar wages and years of education
- own effects sizable and Θ has dominant diagonal
- estimate for $\lambda + \varrho = 0.9$ close to 1
- Next: CES industry structure with $\eta = 0.2$; $\lambda = 0.5$; $\pi = 30\%$

Estimates of Θ and ϱ

Effect	Estimate of $\frac{1}{\lambda}\theta$	Significant?
Own effect	0.73	[t=19.27]
Industry	0.09	[t=1.22]
Geography	0.17	[t=2.24]
Occupation	0.05	[t=2.23]
Wages and Education	0.06	[t=3.33]
$\lambda + \varrho$ (or σ)	0.91	

Quantitative Implications: Effects on Wages

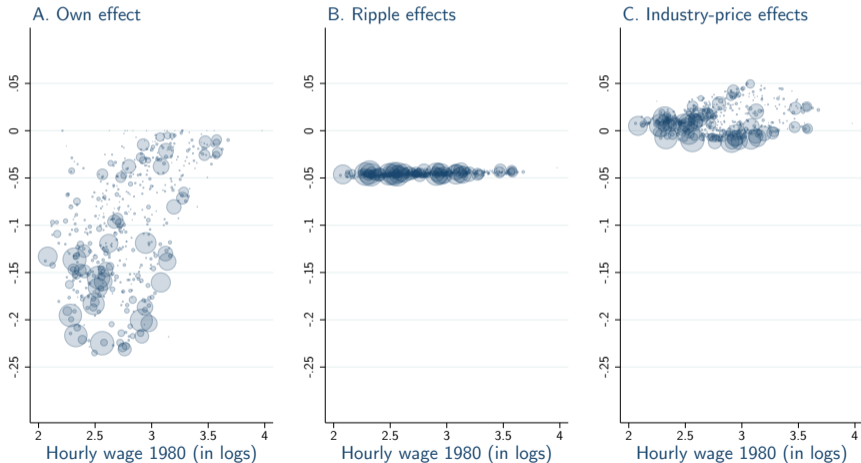


Figure: Effect on wages (not including rise in GDP).

Quantitative Implications: Combined Effect on Wages

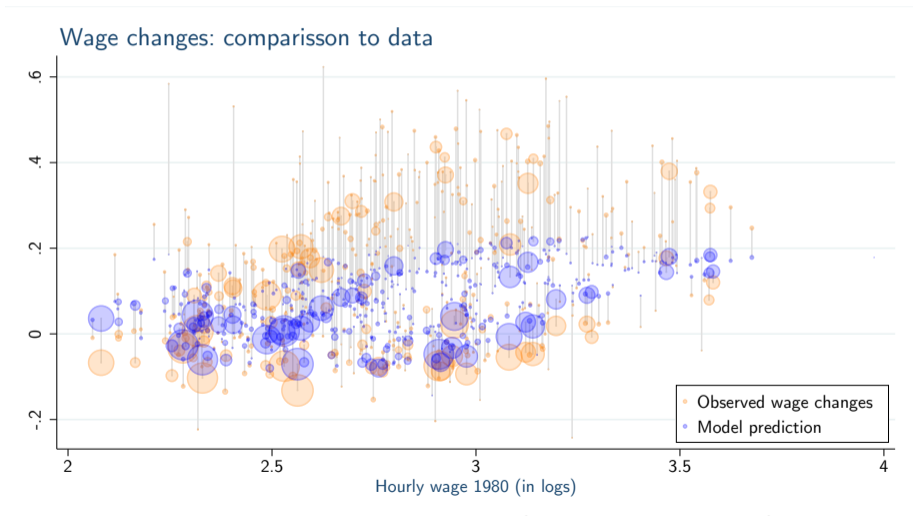


Figure: Combined effect on wages (including rise in GDP).

Quantitative Implications: Groups with Declining Wages

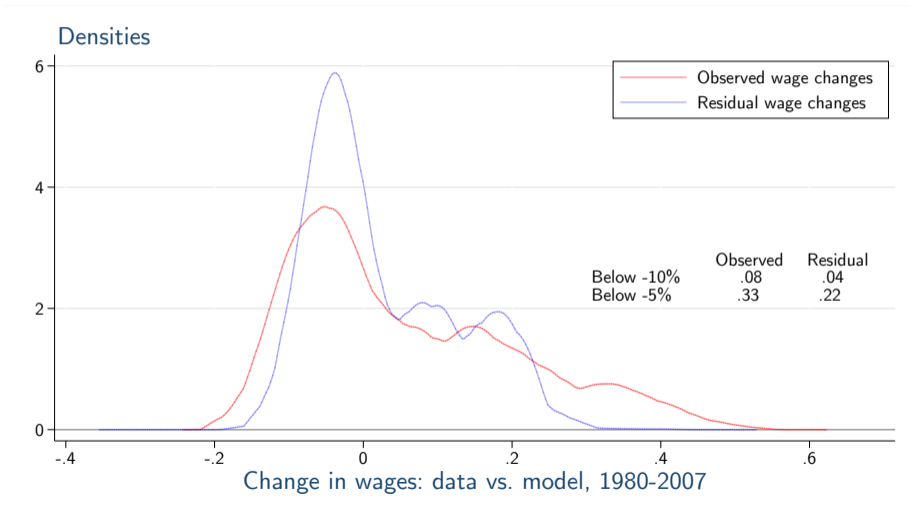


Figure: Observed wage changes and unexplained change in wages.

Quantitative Implications: Summary

Implications of measured task displacement via automation and offshoring:

- Increase in GDP of 20% and average wage of 5%
- TFP increase of 3.3%
- Explains 57% of observed wage changes across groups (48% ignoring industry price changes)
- Explains a third of wage declines below 5% and half of wage declines below 10%
- Explains a third of the rise in college premium and half of rise in postcollege premium
- Explains 0.6 pp decline in share of manufacturing in GDP (1/10th of decline since 1987)

Concluding Remarks:

- technologies that favor displacement of labor via automation or offshoring can have large distributional consequences and bring small productivity gains
- we made this point theoretically in a task-framework, via reduced-form evidence, and through a preliminary quantitative exercise

Work to do:

1. Much more to do regarding estimation of Θ ...
2. Markups? direct measures of technology and estimation
3. Factor-augmenting technologies: bound effect on labor share
4. Repercussions for within-group inequality?

Appendix Model: Formal Definition of Equilibrium

[▶ back to model](#)

Let \mathcal{T}_g denote the set of tasks allocated to labor of type g and \mathcal{T}_k the set of tasks allocated to capital.

Definition (Market equilibrium)

Given a supply of labor $\ell = (\ell_1, \ell_2, \dots, \ell_G)$, a market equilibrium is given by wages $w = (w_1, w_2, \dots, w_G)$, capital production decisions $\{k(x)\}$, and an allocation of tasks to factors $\{\mathcal{T}_k, \mathcal{T}_1, \dots, \mathcal{T}_G\}$, such that:

- the allocation of tasks to factors minimizes the total cost of producing each task;
- the choice of capital maximizes net output;
- the market for capital and labor clears.

Why Focus on Task-Displacing Technologies?

- 1. Rhetorical point**
 - compelling way of thinking about automation and offshoring
 - some tasks can now be automated or offshored; others not
- 2. Mechanism affecting wages**
 - task-displacing techs directly change task-share boundaries
 - large distributional impact that is independent of elast. of subs.
 - effect of other techs mediated by λ , $\lambda + \varrho_g \leq 1$
- 3. Productivity implications**
 - task-displacing techs can have small effects on tfp if $\pi_g \approx 0$
 - other techs: prod gains and distributional effects coupled
 - ▶ example: bounds on productivity
 - ▶ example: SBTC
- 4. Factor shares implications**
 - direct and intuitive effect on labor share that is independent of the elasticities of substitution $\lambda + \varrho_g$

Example I: Bounding Effects on Wage Inequality [▶ back to main](#)

Large G and
uniform rise in
inequality

- observed inequality $d \ln w_g = m_0 + \frac{1}{\lambda} \Theta_g \cdot m_g$, where $m_g \sim U[0, 2\delta]$
- how big is tech change required to explain this rise in inequality?
(assuming no technological regress)

Example I: Bounding Effects on Wage Inequality [▶ back to main](#)

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Via task
deepening

- $d \ln \text{tfp} \geq \delta \cdot s^L / |1 - \lambda|$
- effects through $\lambda \in (0.5, 1) \Rightarrow$ subs across tasks

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Via labor-
augmenting
technologies

- $d \ln \text{tfp} \geq \delta \cdot \sum_g s_g^L / |\sigma_g - 1|$, where $\sigma_g \geq \lambda$
- effects through $\sigma_g \in (1, 2) \Rightarrow$ subs across tasks and within marginal task

Example I: Bounding Effects on Wage Inequality ▶ back to main

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- effects through $\sigma_g \in (1, 2) \Rightarrow$ subs across tasks and within marginal task

Via task
displacement

- $d \ln \text{tfp} \geq \delta \cdot \sum_g s_g^L \cdot \pi_g$, where $\pi_g \geq 0$
- effects through changes in productivity at marginal tasks, π_g

Example II: Unpacking SBTC [▶ back to main](#)

Canonical model

$$d\ln \frac{w_H}{w_L} = -\frac{1}{\sigma_L} \cdot d\ln \frac{H}{L} + \frac{\sigma_L - 1}{\sigma_L} \cdot d\ln \frac{A_H}{A_L}$$

Effects on TFP

$$d\ln \text{tfp} = s^H \cdot d\ln A_H + s^L \cdot d\ln A_L$$

Tight link b/n
inequality,
productivity, and
real wages

- estimate of $\sigma_L = 1.5$
- explains skill premium with $d\ln A_H \geq d\ln A_H/A_L = 10\%$ p.a.
- but this implies $d\ln \text{tfp} \geq 2-3\%$ p.a (vs 1-1.2% in data)
- and $d\ln w_L \geq 1.3-2\%$ p.a (vs -0.2-0.2% in data)

Example II: Unpacking SBTC [▶ back to main](#)

Task model

$$d\ln \frac{w_H}{w_L} = -\frac{1}{\sigma_L} \cdot d\ln \frac{H}{L} + \frac{\sigma_L - 1}{\sigma_L} \cdot d\ln \frac{A_H}{A_L} - \frac{1}{\sigma} \cdot d\ln \Gamma_L^{\text{disp}},$$
$$\sigma_L := (\theta_{HH} - \theta_{LH})^{-1} = (\theta_{LL} - \theta_{HL})^{-1} > \lambda$$

Effects on TFP

$$d\ln \text{tfp} = s^H \cdot d\ln A_H + s^L \cdot d\ln A_L - s^L d\ln \Gamma_L^{\text{disp}} \cdot \pi_L$$

Decoupling of
inequality,
productivity, and
real wages

- suppose $\sigma_L = 1.5$ and $\pi_L = 30\%$
- one can explain skill premium with $d\ln \Gamma_L^{\text{disp}} = -4.5\%$ p.a.
- this implies $d\ln \text{tfp} = 0.45\%$ p.a (vs 1–1.2% in data)
- and $d\ln w_L = -0.55\%$ p.a (vs -0.2–0.2% in data)